Sabbatical Mini-Report #1: Variables, Knowledge, and Competence

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A primary emphasis in the curriculum is mastery of procedures needed to simplify expressions, and solve various types of equations (linear, quadratic, rational, etc). We might be tempted to conclude that students, as a result, have some understanding of "variable".

Fujii (2005) reports on some interesting research to challenge our thinking. In a chapter "Probing Students' Understanding of Variables Through Cognitive Conflict", Fujii discusses what happens when students are shown problems in a different context. These 2 problems are shown below (slightly adapted).

Problem I: Find value(s) for x in the expression $x + x + x = 12$ Circle the letter(s) that is (are) correct.

(a) 2, 5, 5
(b) 10, 1, 1
(c) 4, 4, 4

Problem II: Find value(s) for x and y in the expression $x + y = 16$ Circle the letter(s) that is (are) correct.

(a) 6, 10
(b) 9, 7
(c) 8, 8

Mathematically, of course, the correct answers are (c) for I and (a, b, c) for II. [Note the technical error with the word "expression"; this might be due to the author's challenge of putting technical language into English.]

Fujii conducted research with both American and Japanese students, at different grade levels. The goal is to have both items correct; the percent correct (on both) is given in the following table.

	6^{th}	8^{th}	9^{th}	10^{th}
American school	11.5%	11.5%	5.7%	--
Japanese school	3.7%	10.8%	--	18.1%

The samples here are not large enough to compare countries; the author used one specific school system in each country, so the numbers of students in each case are just in the hundreds. However, the numbers are large enough to indicate that the percent correct on both (in general) is quite low; the author states that this has been consistently the case in similar studies.

> *Note: Since there are 6 possible ways to answer each question (a, b, c, ab, ac, abc), the 'random correct' rate would be $\frac{1}{6}$. The probability of guessing both questions correctly would then be about 3% (0.1667 * 0.1667). At least most results are 'above guessing level'! However, if 70% of the students could answer each one … then the 'both correct' rate would be about 50%.* ☹
> *(These comments assume 'independence' of items; that's probably accurate IF students guess, but not otherwise.)*

It might be discouraging to see the percent actually drop in the American school at the grade level where algebra is actually being studied. If this decline is accurate, it implies that the algebraic instruction is actually reinforcing erroneous ideas about variables – or, at least, it is weakening this understanding in some way.

Fujii proceeds to define 4 "levels of understanding of literal symbols".

Level	Understanding of Variable concept
Level 0	No concept [both problems incorrect]
Level 1	Variable is unspecified [problem II correct; incorrect on I]
Level 2	Variable is a definite (unknown) number [problem I correct; incorrect on II]
Level 3	Variable is definite but unspecified [both problems correct]

The author believes that students proceed through these 4 levels; in other words, students first learn that a variable is 'unspecified' … and then unlearn that idea to accept the 'definite' aspect. Particular effort, according to the author, is needed to get students from Level 2 to Level 3.

Another problem that has been used in this type of research is this one:

Problem III: When is the following true – always, never, or sometimes:

$$L + M + N = L + P + N$$

Studies on this problem indicate that 40% to 74% of students say "never" – which reflects a weak understanding of variable (level 0 or 2 in the chart).

Fujii also conducted interviews on this problem:

Problem IV: $x + \frac{x}{4} = 6 + \frac{x}{4}$

One of the most common statements was "the x on the left must be 6; the x in $\frac{x}{4}$ can be any number". Somehow, students learn to apply procedures and some logic … without understanding the limitations on a variable in one problem.

Fujii's research on the concept of variables is consistent with others; see Trigueros (2003) for example. Many other studies have been done over the last 25 years; those I've read produce similar results.

IMPLICATIONS OF THIS RESEARCH:

It's unclear if "understand concept of variable" is really part of the accepted (intended) curriculum. However, IF we do wish our students to understand "variable", then it is clear we need to do more than assess whether students can simplify and solve. Many of the students involved in Fujii's research were succeeding in their algebra classes; this success did not translate into better understanding of 'variable'.

Of course, our curricular materials are not especially strong on developing the understanding directly. Perhaps this seems "too theoretical". I would suggest that there is nothing more central to algebra than the concept of variable. Instructors might need to create opportunities for students to develop a deeper understanding of variable.

My own plans are to include some of these type of problems (above) on worksheets and/or writing assignments. Later, I might include some test items on 'variable'.

References:

Fujii, Toshiakaira	Probing Student's Understanding of Variables Through Cognitive Conflict	in Proceedings of the 2003 Joint Meeting o PME and PMENA

["PME" stands for "Psychology of Mathematics Education"; "PMENA" stands for "Psychology of Mathematics Education North America"]
This chapter is also available online (as of November 21, 2006):
Online version: http://onlinedb.terc.edu/PME2003/PDF/Plen5fujii.pdf

Trigueros, Maria; Ursini, Sonia	First-Year Undergraduates' Difficulties in Working with Different Uses of Variable	in Research in Collegiate Mathematics Education V (edited by Annie Seldon, et al